Optimal-Observable Analysis of Possible New Physics Using the b-quark in $\gamma\gamma \rightarrow t\bar{t} \rightarrow bX$

Bohdan GRZADKOWSKI [1), a)], Zenrō HIOKI [2), b)],

Kazumasa OHKUMA [3), c)] and José WUDKA [4), d)]

1) *Institute of Theoretical Physics, Warsaw University*
Hoża 69, PL-00-681 Warsaw, Poland
and
CERN, Department of Physics
Theory Division
1211 Geneva 23, Switzerland

2) *Institute of Theoretical Physics, University of Tokushima*
Tokushima 770-8502, Japan

3) *Department of Information Science, Fukui University of Technology*
Fukui 910-8505, Japan

4) *Department of Physics, University of California*
Riverside, CA 92521-0413, USA

ABSTRACT

We study possible anomalous top-quark couplings generated by $SU(2) \times U(1)$ gauge-invariant dimension-6 effective operators, using the final b-quark momentum distribution in $\gamma\gamma \rightarrow t\bar{t} \rightarrow bX$. Taking into account non-standard $t\bar{t}\gamma$, tbW and $\gamma\gamma H$ couplings, we perform an optimal-observable analysis in order to estimate the precision for the determination of all relevant non-standard couplings.

[a)] E-mail address: `bohdan.grzadkowski@fuw.edu.pl`
[b)] E-mail address: `hioki@ias.tokushima-u.ac.jp`
[c)] E-mail address: `ohkuma@ccmails.fukui-ut.ac.jp`
[d)] E-mail address: `jose.wudka@ucr.edu`

1. Introduction

Linear colliders of e^+e^- are expected to work as top-quark factories, and therefore a lot of attention has been paid to study possible non-standard top-quark interactions through $e\bar{e} \to t\bar{t}$ (see, for instance, [1, 2] and their reference lists). An interesting option for such e^+e^- machines could be that of photon–photon collisions, where initial energetic photons are produced through electron and laser-light backward scatterings [3, 4].

This type of colliders presents remarkable advantages for the study of CP violation. In the case of $e\bar{e}$ collisions, the only initial states that are relevant are CP-even states $|e_{L/R}\bar{e}_{R/L}\rangle$ under the usual assumption that the electron mass can be neglected and that the leading contributions to $t\bar{t}$ production come from s-channel vector-boson exchanges. Therefore, all CP-violating observables must be constructed from final-particle momenta/polarizations. In contrast, a $\gamma\gamma$ collider offers a unique possibility of preparing the polarization of the incident-photon beams, which can be used to construct CP-violating asymmetries without relying on final-state information.

This is why a number of authors have considered top-quark production and decays in $\gamma\gamma$ collisions in order to study i) Higgs-boson couplings to the top quark and photon [5]–[11], or ii) anomalous top-quark couplings to the photon [12]–[14]. However, what is supposed to be observed in real experiments is combined signals that originate both from the process of top-quark production and, *in addition*, from its decays. Therefore, in our latest article [15] we considered $\gamma\gamma \to t\bar{t} \to \ell^+ X$, including all possible non-standard interactions together (production and decay), and performed a comprehensive analysis as model-independently as possible within the effective-Lagrangian framework of Buchmüller and Wyler [16].

In this letter, we will carry out an optimal-observable (OO) analysis, using the final b-quark momentum distribution, as a complementary work to [15]. What we have to do for this purpose is similar to what has been done in [15]. However, in the case of the bX final state, we can expect to obtain independent and valuable information since there is no branching-ratio suppression for $t \to bW$, in contrast to the analysis with the final lepton. One might say that using the final b-quark

distribution is not that effective, since the determination of the b-quark momentum is more challenging than that of charged leptons. However, in any case, it is crucial to tag the final b-quark efficiently in order to distinguish the top-quark production from the main background of W^+W^- production [17]. That is, we cannot study top-quark events without good information on the final b-quark, which makes our analysis realistic.

2. Framework

We use the effective low-energy Lagrangian [16, 18] to describe possible new-physics effects. Following this approach, we consider the Standard-Model (SM) Lagrangian modified by the addition of a series of $SU(2) \times U(1)$ gauge-invariant operators $\mathcal{O}_i$ whose coefficients parameterize the low-energy effects of the underlying high-scale physics.

Since the detailed description of this framework was presented in [15], we only mention here that the largest contribution comes from dimension-6 operators, and that these lead to the following Feynman rules for on-shell photons, which are necessary for our calculations:

(1) CP-conserving $t\bar{t}\gamma$ vertex

$$\sqrt{2} v \alpha_{\gamma 1} \not{k} \gamma_\mu / \Lambda^2, \tag{1}$$

(2) CP-violating $t\bar{t}\gamma$ vertex

$$i\sqrt{2} v \alpha_{\gamma 2} \not{k} \gamma_\mu \gamma_5 / \Lambda^2, \tag{2}$$

(3) CP-conserving $\gamma\gamma H$ vertex

$$-4v\alpha_{h1} \left[(k_1 k_2) g_{\mu\nu} - k_{1\nu} k_{2\mu} \right] / \Lambda^2, \tag{3}$$

(4) CP-violating $\gamma\gamma H$ vertex

$$8v\alpha_{h2} \, k_1^\rho k_2^\sigma \epsilon_{\rho\sigma\mu\nu} / \Lambda^2, \tag{4}$$

where $v \sim 250$ GeV, k and $k_{1,2}$ are incoming photon momenta, and $\alpha_{\gamma 1, \gamma 2, h1, h2}$ are defined as

$$\alpha_{\gamma 1} \equiv \sin\theta_W \mathrm{Re}(\alpha_{uW}) + \cos\theta_W \mathrm{Re}(\alpha'_{uB}), \tag{5}$$

$$\alpha_{\gamma 2} \equiv \sin\theta_W \mathrm{Im}(\alpha_{uW}) + \cos\theta_W \mathrm{Im}(\alpha'_{uB}), \tag{6}$$

$$\alpha_{h1} \equiv \sin^2\theta_W \mathrm{Re}(\alpha_{\varphi W}) + \cos^2\theta_W \mathrm{Re}(\alpha_{\varphi B}) - 2\sin\theta_W \cos\theta_W \mathrm{Re}(\alpha_{WB}), \tag{7}$$

$$\alpha_{h2} \equiv \sin^2\theta_W \mathrm{Re}(\alpha_{\varphi\tilde{W}}) + \cos^2\theta_W \mathrm{Re}(\alpha_{\varphi\tilde{B}}) - \sin\theta_W \cos\theta_W \mathrm{Re}(\alpha_{\tilde{W}B}), \tag{8}$$

α_i and α'_j being the coefficients of $\mathcal{O}_i$ and $\mathcal{O}'_j$ ($i = uW$, φW, φB, WB, $\varphi\tilde{W}$, $\varphi\tilde{B}$, $\tilde{W}B$ and $j = uB$) respectively, and θ_W the Weinberg angle. It will be helpful to note that the SM $f\bar{f}\gamma$ coupling in our scheme is given by $eQ_f\gamma_\mu$, where e is the proton charge and Q_f is f's electric charge in e unit (e.g. $Q_u = 2/3$).

The top-quark decay vertex is also affected by some dim-6 operators. For the on-mass-shell W boson it will be sufficient to consider just the following tbW amplitude when m_b is neglected:

$$\Gamma^\mu_{Wtb} = -\frac{g}{2\sqrt{2}} \bar{u}(p_b)\left[\gamma^\mu(1 - \gamma_5) - \frac{i\sigma^{\mu\nu}k_\nu}{M_W} f_2^R(1 + \gamma_5) \right]u(p_t), \tag{9}$$

where f_2^R is given by

$$f_2^R = -\frac{v}{\Lambda^2}M_W\left[\frac{4}{g}\alpha_{uW} + \frac{1}{2}\alpha_{Du} \right], \tag{10}$$

with α_{Du} the coefficient of the operator $\mathcal{O}_{Du}$ [#1]. On the other hand, the $\nu\ell W$ vertex is assumed to receive negligible contributions from physics beyond the SM.

Finally, the initial-state polarizations are characterized by the initial electron and positron longitudinal polarizations P_e and $P_{\bar{e}}$, the average helicities of the initial-laser photons P_γ and $P_{\tilde{\gamma}}$, and their maximum average linear polarizations P_t and $P_{\tilde{t}}$ with the azimuthal angles φ and $\tilde{\varphi}$ (defined in the same way as in [3]). The polarizations $P_{\gamma,t}$ and $P_{\tilde{\gamma},\tilde{t}}$ have to satisfy

$$0 \leq P_\gamma^2 + P_t^2 \leq 1, \qquad 0 \leq P_{\tilde{\gamma}}^2 + P_{\tilde{t}}^2 \leq 1. \tag{11}$$

3. Optimal-observable analysis

The calculation of the cross section is straightforward; to derive distributions of secondary fermions we have applied the Kawasaki–Shirafuji–Tsai technique [19] with FORM [20] used for the necessary algebraic manipulations. We neglect contributions that are quadratic in non-standard interactions and treat the decaying

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[#1]Note that there is another potential source of contribution to f_2^R, which may come from $\mathcal{O}_{\bar{D}u}$. However, this operator could be eliminated using equations of motion; therefore, it is neglected hereafter. We thank Ilya Ginzburg for pointing this to us.

t and W as on-shell particles; therefore the angular-energy distribution of the b quark in the $e\bar{e}$ CM frame can be expressed as

$$\frac{d\sigma}{dE_b d\cos\theta_b} = f_{\mathrm{SM}}(E_b, \cos\theta_b) + \alpha_{\gamma 1} f_{\gamma 1}(E_b, \cos\theta_b) + \alpha_{\gamma 2} f_{\gamma 2}(E_b, \cos\theta_b)$$

$$+ \alpha_{h1} f_{h1}(E_b, \cos\theta_b) + \alpha_{h2} f_{h2}(E_b, \cos\theta_b) + \alpha_d f_d(E_b, \cos\theta_b), \qquad (12)$$

where $f_i(E_b, \cos\theta_b)$ are calculable functions; f_{SM} denotes the standard-model contribution, $f_{\gamma 1, \gamma 2}$ describe, respectively, the anomalous CP-conserving and CP-violating $t\bar{t}\gamma$-vertices contributions, $f_{h1,h2}$ those generated by the anomalous CP-conserving and CP-violating $\gamma\gamma H$-vertices, and f_d that by the anomalous tbW-vertex with

$$\alpha_d = \mathrm{Re}(f_2^R).$$

Their analytical form is however too long to be presented in this letter.

In order to apply the OO technique, we first have to calculate the following matrix elements using the weighting functions $f_i(E_b, \cos\theta_b)$ defined in eq. (12):

$$\mathcal{M}_{ij} = \int dE_b d\cos\theta_b\, f_i(E_b, \cos\theta_b) f_j(E_b, \cos\theta_b) / f_{\mathrm{SM}}(E_b, \cos\theta_b), \qquad (13)$$

and its inverse matrix X_{ij}, where $i, j = 1, \cdots, 6$ correspond to SM, $\gamma 1$, $\gamma 2$, $h1$, $h2$ and d respectively. Then, according to [21], the expected statistical uncertainty for the measurements of α_i is given by

$$|\Delta\alpha_i| = \sqrt{I_0 X_{ii}/N_b}, \qquad (14)$$

where

$$I_0 \equiv \int dE_b d\cos\theta_b\, f_{\mathrm{SM}}(E_b, \cos\theta_b)$$

and N_b is the total number of collected events.

Inverting the matrix $\mathcal{M}$, we have noticed that the numerical results for X_{ij} are often unstable: even a tiny variation of $\mathcal{M}_{ij}$ changes X_{ij} significantly. This indicates that some of f_i have similar shapes[#2] and therefore their coefficients cannot be disentangled easily. Indeed, we already encountered a similar trouble in our latest analysis using final leptons [15]. It is not surprising that we meet this

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[#2]Note that if two f_i functions were proportional to each other, then the matrix $\mathcal{M}_{ij}$ would have a vanishing determinant, and therefore its inverse X_{ij} could not be determined.

problem again here, since the main structure of the cross section is determined by that of $\gamma\gamma \to t\bar{t}$ for both processes.

The presence of such instability forces us to refrain from determining all the couplings at once through this process alone. Therefore, hereafter, we assume that some of α_i's have been measured in other processes (e.g. in $e\bar{e} \to t\bar{t} \to \ell^\pm X$). Fortunately, however, we obtain some complementary information on coupling constants, which was not available in our previous analysis [15], where only leptonic distributions were employed.

Below we list all the elements of $\mathcal{M}\ (= \mathcal{M}^T)$, which were computed for

$$\sqrt{s_{e\bar{e}}} = 500 \text{ GeV} \quad \text{and} \quad \Lambda = 1 \text{ TeV}. \tag{15}$$

(1) Linear polarization

We chose the following values as typical linear polarizations: $P_e = P_{\bar{e}} = 1$, $P_t = P_{\tilde{t}} = P_\gamma = P_{\tilde{\gamma}} = 1/\sqrt{2}$ and $\chi(\equiv \varphi - \tilde{\varphi}) = \pi/4$, where φ and $\tilde{\varphi}$ are the azimuthal angles of P_t and $P_{\tilde{t}}$. They are the same polarizations as those we used in [15].

1-1) $m_H = 100$ GeV

$$
\begin{aligned}
&\mathcal{M}_{11} = 0.368 \times 10^2, &&\mathcal{M}_{12} = 0.787 \times 10^2, &&\mathcal{M}_{13} = -0.323 \times 10^1, \\
&\mathcal{M}_{14} = -0.145 \times 10^2, &&\mathcal{M}_{15} = -0.153 \times 10^1, &&\mathcal{M}_{16} = 0, \\
&\mathcal{M}_{22} = 0.169 \times 10^3, &&\mathcal{M}_{23} = -0.699 \times 10^1, &&\mathcal{M}_{24} = -0.299 \times 10^2, \\
&\mathcal{M}_{25} = -0.331 \times 10^1, &&\mathcal{M}_{26} = 0.277 \times 10^1, &&\mathcal{M}_{33} = 0.352, \\
&\mathcal{M}_{34} = 0.122 \times 10^1, &&\mathcal{M}_{35} = 0.182, &&\mathcal{M}_{36} = -0.454, \\
&\mathcal{M}_{44} = 0.681 \times 10^1, &&\mathcal{M}_{45} = 0.583, &&\mathcal{M}_{46} = 0.271 \times 10^1, \\
&\mathcal{M}_{55} = 0.987 \times 10^{-1}, &&\mathcal{M}_{56} = -0.281, &&\mathcal{M}_{66} = 0.866 \times 10^1.
\end{aligned}
\tag{16}
$$

1-2) $m_H = 300$ GeV

$$
\begin{aligned}
&\mathcal{M}_{11} = 0.368 \times 10^2, &&\mathcal{M}_{12} = 0.787 \times 10^2, &&\mathcal{M}_{13} = -0.323 \times 10^1, \\
&\mathcal{M}_{14} = -0.359 \times 10^2, &&\mathcal{M}_{15} = -0.691 \times 10^1, &&\mathcal{M}_{16} = 0, \\
&\mathcal{M}_{22} = 0.169 \times 10^3, &&\mathcal{M}_{23} = -0.699 \times 10^1, &&\mathcal{M}_{24} = -0.742 \times 10^2, \\
&\mathcal{M}_{25} = -0.146 \times 10^2, &&\mathcal{M}_{26} = 0.277 \times 10^1, &&\mathcal{M}_{33} = 0.352, \\
&\mathcal{M}_{34} = 0.298 \times 10^1, &&\mathcal{M}_{35} = 0.681, &&\mathcal{M}_{36} = -0.454, \\
&\mathcal{M}_{44} = 0.421 \times 10^2, &&\mathcal{M}_{45} = 0.725 \times 10^1, &&\mathcal{M}_{46} = 0.711 \times 10^1, \\
&\mathcal{M}_{55} = 0.146 \times 10^1, &&\mathcal{M}_{56} = 0.143, &&\mathcal{M}_{66} = 0.866 \times 10^1.
\end{aligned}
\tag{17}
$$

1-3) $m_H = 500$ GeV

$$
\begin{aligned}
&\mathcal{M}_{11} = 0.368 \times 10^2, \quad &&\mathcal{M}_{12} = 0.787 \times 10^2, \quad &&\mathcal{M}_{13} = -0.323 \times 10^1, \\
&\mathcal{M}_{14} = 0.170 \times 10^2, \quad &&\mathcal{M}_{15} = -0.101 \times 10^2, \quad &&\mathcal{M}_{16} = 0, \\
&\mathcal{M}_{22} = 0.169 \times 10^3, \quad &&\mathcal{M}_{23} = -0.699 \times 10^1, \quad &&\mathcal{M}_{24} = 0.352 \times 10^2, \\
&\mathcal{M}_{25} = -0.206 \times 10^2, \quad &&\mathcal{M}_{26} = 0.277 \times 10^1, \quad &&\mathcal{M}_{33} = 0.352, \\
&\mathcal{M}_{34} = -0.148 \times 10^1, \quad &&\mathcal{M}_{35} = 0.809, \quad &&\mathcal{M}_{36} = -0.454, \\
&\mathcal{M}_{44} = 0.935 \times 10^1, \quad &&\mathcal{M}_{45} = -0.579 \times 10^1, \quad &&\mathcal{M}_{46} = -0.283 \times 10^1, \\
&\mathcal{M}_{55} = 0.369 \times 10^1, \quad &&\mathcal{M}_{56} = 0.253 \times 10^1, \quad &&\mathcal{M}_{66} = 0.866 \times 10^1.
\end{aligned}
\tag{18}
$$

(2) Circular polarization

We took the following values as circular-polarization parameters: $P_e = P_{\bar{e}} = P_\gamma = P_{\tilde{\gamma}} = 1$, which were also used in [15].

2-1) $m_H = 100$ GeV

$$
\begin{aligned}
&\mathcal{M}_{11} = 0.209 \times 10^2, \quad &&\mathcal{M}_{12} = 0.454 \times 10^2, \quad &&\mathcal{M}_{13} = 0, \\
&\mathcal{M}_{14} = -0.690 \times 10^1, \quad &&\mathcal{M}_{15} = -0.109 \times 10^{-3}, \quad &&\mathcal{M}_{16} = 0, \\
&\mathcal{M}_{22} = 0.988 \times 10^2, \quad &&\mathcal{M}_{23} = 0, \quad &&\mathcal{M}_{24} = -0.144 \times 10^2, \\
&\mathcal{M}_{25} = -0.227 \times 10^{-3}, \quad &&\mathcal{M}_{26} = 0.126 \times 10^1, \quad &&\mathcal{M}_{33} = 0, \\
&\mathcal{M}_{34} = 0, \quad &&\mathcal{M}_{35} = 0, \quad &&\mathcal{M}_{36} = 0, \\
&\mathcal{M}_{44} = 0.284 \times 10^1, \quad &&\mathcal{M}_{45} = 0.457 \times 10^{-4}, \quad &&\mathcal{M}_{46} = 0.133 \times 10^1, \\
&\mathcal{M}_{55} = 0.739 \times 10^{-9}, \quad &&\mathcal{M}_{56} = 0.243 \times 10^{-4}, \quad &&\mathcal{M}_{66} = 0.393 \times 10^1.
\end{aligned}
\tag{19}
$$

2-2) $m_H = 300$ GeV

$$
\begin{aligned}
&\mathcal{M}_{11} = 0.209 \times 10^2, \quad &&\mathcal{M}_{12} = 0.454 \times 10^2, \quad &&\mathcal{M}_{13} = 0, \\
&\mathcal{M}_{14} = -0.178 \times 10^2, \quad &&\mathcal{M}_{15} = -0.177 \times 10^1, \quad &&\mathcal{M}_{16} = 0, \\
&\mathcal{M}_{22} = 0.988 \times 10^2, \quad &&\mathcal{M}_{23} = 0, \quad &&\mathcal{M}_{24} = -0.373 \times 10^2, \\
&\mathcal{M}_{25} = -0.368 \times 10^1, \quad &&\mathcal{M}_{26} = 0.126 \times 10^1, \quad &&\mathcal{M}_{33} = 0, \\
&\mathcal{M}_{34} = 0, \quad &&\mathcal{M}_{35} = 0, \quad &&\mathcal{M}_{36} = 0, \\
&\mathcal{M}_{44} = 0.191 \times 10^2, \quad &&\mathcal{M}_{45} = 0.193 \times 10^1, \quad &&\mathcal{M}_{46} = 0.360 \times 10^1, \\
&\mathcal{M}_{55} = 0.198, \quad &&\mathcal{M}_{56} = 0.419, \quad &&\mathcal{M}_{66} = 0.393 \times 10^1.
\end{aligned}
\tag{20}
$$

2-3) $m_H = 500$ GeV

$$
\begin{aligned}
&\mathcal{M}_{11} = 0.209 \times 10^2, \quad &&\mathcal{M}_{12} = 0.454 \times 10^2, \quad &&\mathcal{M}_{13} = 0, \\
&\mathcal{M}_{14} = 0.762 \times 10^1, \quad &&\mathcal{M}_{15} = -0.502 \times 10^1, \quad &&\mathcal{M}_{16} = 0, \\
&\mathcal{M}_{22} = 0.988 \times 10^2, \quad &&\mathcal{M}_{23} = 0, \quad &&\mathcal{M}_{24} = 0.159 \times 10^2, \\
&\mathcal{M}_{25} = -0.105 \times 10^2, \quad &&\mathcal{M}_{26} = 0.126 \times 10^1, \quad &&\mathcal{M}_{33} = 0, \\
&\mathcal{M}_{34} = 0, \quad &&\mathcal{M}_{35} = 0, \quad &&\mathcal{M}_{36} = 0, \\
&\mathcal{M}_{44} = 0.347 \times 10^1, \quad &&\mathcal{M}_{45} = -0.233 \times 10^1, \quad &&\mathcal{M}_{46} = -0.138 \times 10^1, \\
&\mathcal{M}_{55} = 0.158 \times 10^1, \quad &&\mathcal{M}_{56} = 0.103 \times 10^1, \quad &&\mathcal{M}_{66} = 0.393 \times 10^1.
\end{aligned}
\tag{21}
$$

All the elements $\mathcal{M}_{ij}$ above are given in units of fb. In these results, the third components of $\mathcal{M}$ for the circular polarization vanish [12]. This is common for

analyses using the final lepton and the final b-quark. Also, as in the leptonic case, $\mathcal{M}_{16} = 0$. This time, however, it is not because of the decoupling, which holds for the lepton production [22], but simply because the tbW vertex cannot contribute to the total cross section of $\gamma\gamma \to t\bar{t} \to bX$, since

$$\sigma_{\text{tot}}(\gamma\gamma \to t\bar{t} \to bX) = Br(t \to bX)\sigma_{\text{tot}}(\gamma\gamma \to t\bar{t})$$

and $Br(t \to bX) = 1$, whatever anomalous terms are added to the tbW coupling as long as we assume that a top quark always decays through $t \to bW$.

When estimating the statistical uncertainty in simultaneous measurements, e.g. of $\alpha_{\gamma 1}$ and α_{h1} (assuming all other coefficients are known), we need only the components with indices 1, 2 and 4. Let us express the resultant uncertainties as $\Delta\alpha_{\gamma 1}^{[3]}$ and $\Delta\alpha_{h1}^{[3]}$, where "3" shows that we used the input $\mathcal{M}_{ij}$, keeping three decimal places. In order to see how stable the results are, we also computed $\Delta\alpha_{\gamma 1}^{[2]}$ and $\Delta\alpha_{h1}^{[2]}$ by rounding $\mathcal{M}_{ij}$ off to two decimal places. Then, if both of the deviations $|\Delta\alpha_{\gamma 1,h1}^{[3]} - \Delta\alpha_{\gamma 1,h1}^{[2]}|/\Delta\alpha_{\gamma 1,h1}^{[3]}$ are less than 10%, we accept the result as a stable solution.

Although we did not find any stable solution in the three-parameter analysis, we did find some solutions in a two-parameter analysis; for those, the numerical results are presented below. According to the above criterion, the uncertainties for the following standard deviations $\Delta\alpha_i$ are limited to 10%:

1) Linear polarization

● Independent of m_H

$$\Delta\alpha_{\gamma 2} = 29/\sqrt{N_b}, \quad \Delta\alpha_d = 2.6/\sqrt{N_b}, \tag{22}$$

● $m_H = 100$ GeV

$$\Delta\alpha_{h2} = 38/\sqrt{N_b}, \quad \Delta\alpha_d = 2.4/\sqrt{N_b}, \tag{23}$$

● $m_H = 300$ GeV

$$\Delta\alpha_{\gamma 2} = 24/\sqrt{N_b}, \quad \Delta\alpha_{h1} = 2.4/\sqrt{N_b}, \tag{24}$$

$$\Delta\alpha_{h1} = 5.4/\sqrt{N_b}, \quad \Delta\alpha_d = 4.9/\sqrt{N_b}, \tag{25}$$

- $m_H = 500$ GeV

$$\Delta\alpha_{\gamma2} = 23/\sqrt{N_b}, \qquad \Delta\alpha_{h1} = 5.0/\sqrt{N_b}, \tag{26}$$

$$\Delta\alpha_{h1} = 18/\sqrt{N_b}, \qquad \Delta\alpha_{h2} = 22/\sqrt{N_b}, \tag{27}$$

$$\Delta\alpha_{h1} = 8.0/\sqrt{N_b}, \qquad \Delta\alpha_{d} = 3.3/\sqrt{N_b}, \tag{28}$$

where $N_b \simeq 18400$ for a luminosity of $L_{e\bar{e}}^{\text{eff}} \equiv \epsilon L_{e\bar{e}} = 500$ fb^{-1} with ϵ being the relevant detection efficiency and $L_{e\bar{e}}$ being the integrated luminosity.[#3]

2) Circular polarization

- $m_H = 100$ GeV

$$\Delta\alpha_{h1} = 14/\sqrt{N_b}, \qquad \Delta\alpha_{d} = 5.2/\sqrt{N_b}, \tag{29}$$

- $m_H = 500$ GeV

$$\Delta\alpha_{h1} = 10/\sqrt{N_b}, \qquad \Delta\alpha_{d} = 4.2/\sqrt{N_b}, \tag{30}$$

where $N_b \simeq 10500$ for $L_{e\bar{e}}^{\text{eff}} = 500$ fb^{-1}.

It is worth while to compare estimations of sensitivities obtained here for the bX final state, with those found in [15] in the case of the $\ell^{\pm}X$ final state. Unfortunately, here, we did not find any stable solution that would allow for a determination of $\alpha_{\gamma1}$; the same was also observed for $\ell^{\pm}X$. We therefore have to look for other suitable processes to determine this parameter. The precision of $\alpha_{\gamma2}$ is not very good either, but it is still much better than in the case of the lepton analysis. On the other hand, we can see that analyzing the b-quark process with linearly polarized beams enables us to estimate some $\Delta\alpha_i$ that were unstable in the lepton analysis, i.e. cases (24) and (27). One of them, eq. (27), is especially useful to probe the CP properties of heavy Higgs bosons through the determination of α_{h1} and α_{h2}. As for the determination of α_d, the $\ell^{\pm}X$ final state seems to be more appropriate. These comparisons show that both final states (bX and $\ell^{\pm}X$) provide complementary information and should therefore be included in a complete analysis.

[#3]Hereafter we use the tree-level SM formula for computing N_b, therefore, below we have the same N_b for different m_H. Also, for illustration, we assumed $L_{e\bar{e}} = 500$ fb^{-1} (adopting $\epsilon = 1$) as the standard reference point. However, one should not forget that tagging a b-quark jet including its charge identification is harder than that of a lepton.

The above results are for $\Lambda = 1$ TeV. When one takes the new-physics scale to be $\Lambda' = \lambda\Lambda$, then all the above results $(\Delta\alpha_i)$ are replaced with $\Delta\alpha_i/\lambda^2$, which means that the right-hand sides of eqs. (22)–(30) are multiplied by λ^2.

4. Summary and Discussion

We studied here beyond the SM effects in the process $\gamma\gamma \to t\bar{t} \to bX$ for arbitrarily polarized photon beams, taking advantage of the fact that polarizations of the incoming-photon beams can be controlled. Non-standard interactions have been parameterized through dim-6 local and gauge-symmetric effective operators à la Buchmüller and Wyler [16]. Assuming that those new-physics effects are small, we have kept only terms linear in corrections to the SM tree-level vertices.

We applied the optimal-observable technique to final b-quark distributions, and estimated statistical significances of measuring each (allowed by the gauge invariance) non-standard parameter. Unfortunately, we had to conclude that it is never possible to determine all the independent non-standard parameters at once through $\gamma\gamma \to t\bar{t} \to bX$ alone. However, we still would be able to perform a useful analysis if we could utilize the complementary information collected in other independent processes.

Comments on the background are here in order. The most serious background is W-boson pair productions. Indeed, its total cross section could be 300 times larger than $\sigma_{\mathrm{tot}}(t\bar{t})$. Fortunately, however, a simulation study has shown that $t\bar{t}$ events can be selected with a signal-to-background ratio of 10 by imposing appropriate invariant-mass constraints on the final-particle momenta [17]. There, an efficient b-quark tagging is crucial, which is a basic assumption in the analysis presented here.

Some non-standard couplings, which should be determined here, could also be studied in the standard e^+e^- option of a linear collider. Therefore, it is worth while to compare the potential power of the two options. As far as the parameter $\alpha_{\gamma 1}$ is concerned, the $\gamma\gamma$ collider does not allow for its determination, while it could be determined at e^+e^-. The second $t\bar{t}\gamma$ coupling $\alpha_{\gamma 2}$, which is proportional to the real part of the top-quark electric dipole moment,[#4] can be measured here. It

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[#4]See [23] taking into account that the operators $\mathcal{O}_{uB}$, $\mathcal{O}_{qB}$ and $\mathcal{O}_{qW}$ are redundant.

should be recalled that energy and polar-angle distributions of leptons and b-quarks in e^+e^- colliders are sensitive only to the imaginary part of the electric dipole moment,[♯5] while here the real part could be determined. For the measurement of $\gamma\gamma H$ couplings, e^+e^- colliders are, of course, useless, while here, for the bX final state both α_{h1} and α_{h2} could be measured. In the case of the decay form factor α_d measurement, the e^+e^- option seems to be a little more advantageous, especially if e^+e^- polarization can be tuned appropriately [25].

It should be emphasized here that the effective-operator strategy adopted in this article is valid only for $\Lambda \gg v \simeq 250$ GeV, in contrast to the analysis of $e^+e^- \to t\bar{t} \to \ell^\pm X$ performed in [22] and [25] for example. Should the reaction $\gamma\gamma \to t\bar{t} \to bX$ exhibit a deviation from the SM predictions that cannot be described properly within this framework, we would have an indication of low-energy beyond-the-SM physics, e.g. two-Higgs-doublet models with new scalar degrees of freedom of relatively low mass scale.

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[♯5]However, it should be emphasized that there exist observables sensitive also to the real part of the top-quark electric dipole moment, see [24].

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